LONDON - PARIS BACHELIER WORKSHOP ON MATHEMATICAL FINANCE

PARIS, 29-30 SEPTEMBER 2016

SCHEDULE

Thursday, 29 September 2016
Fédération Bancaire Française, Auditorium, 18 rue la Fayette, 75009 Paris.

14:00 - 14:30: Welcome and registration.
14:30 - 15:00: Sergio Pulido, *The Jacobi Stochastic Volatility Model*.
15:00 - 15:30: Paolo Pigato, *Statistical Estimation of the Oscillating Brownian Motion and Application to Volatility Modeling*.
15:30 - 16:00: Blanka Horvath, *Aspects of Asymptotic Expansions in Fractional Volatility Models*.
16:00 - 16:30: Coffee break.
16:30 - 17:00: Anna Aksamit, *Additional Information and Pricing-Hedging Duality in Robust Framework*.
17:00 - 17:30: Paul Gassiat, *On Root's Solution to the Skorokhod Embedding Problem*.
17:30 - 18:00: Stefano De Marco, *Asymptotics and Calibration for American Options*.
18.00 - 19.00: Welcome reception.

Friday, 30 September 2016
Institut Henri Poincaré, Amphithéâtre Hermite, 11 rue Pierre et Marie Curie, 75005 Paris.

9:00 - 9:30: Mohamed Mrad, *Convergence Rate of Strong Approximations of Compound Random Maps: An Application to Utility SPDEs*.
9:30 - 10:00: Umut Cetin, *Linear Inverse Problems for Markov Processes and their Regularization*.
10:00 - 10:30: Jean-François Chassagneux, *Cubature Method to Solve BSDEs: Error Expansion and Complexity Control*.
10:30 - 11:00: Coffee break.
11:00 - 11:30: Eyal Neuman, *Optimal Portfolio Liquidation in Target Zone Models and Catalytic Superprocesses*.
11:30 - 12:00: Arne Lokka, *Optimal Liquidation Trajectories for the Algren-Chriss Model with Lévy Processes*.
12:00 - 12:30: Roxana Dumitrescu, *Game Options in an Imperfect Market with Default*.
12:30 - 14:30: Lunch and **poster session**.
14:30 - 15:00: Daniel Schwarz, *Integral Representation of Martingales in Mathematical Finance*.
15:00 - 15:30: Lane Hughston, *Lévy-Vasicek Models and the Long-Bond Return Process*.
15:30 - 16:00: Cristin Buescu, *Funding Inclusive Valuation as Modified Option Pricing*.
16:00 - 16:30: Thibaut Mastrolia, *A Tale of a Principal and Many Agents*.
16:30 - 18:00: Discussion session and coffee break.

Organizing Committee (in collaboration with the Bachelier Paris group):
Bruno Bouchard (Université Paris Dauphine), Luciano Campi (London School of Economics), Claudio Fontana (Université Paris Diderot), Emmanuel Gobet (École Polytechnique), Lane Hughston (Brunel University London), Antoine Jacquier (Imperial College London), Teemu Pennanen (King's College London), Johannes Ruf (University College London), Xiaolu Tan (Université Paris Dauphine).

ABSTRACTS OF THE TALKS

Anna AKSAMIT, University of Oxford.
Title: *Additional Information and Pricing-Hedging Duality in Robust Framework*.
Abstract: In robust approach, instead of choosing one model, one considers superhedging simultaneously under a family of models, or pathwise on the set of feasible trajectories. Usually in the literature the focus is on the natural filtration $\mathbb{F}$ of the price process. Here we extend that to a general filtration $\mathbb{G}$ including the natural filtration of the price process $\mathbb{F} \subset \mathbb{G}$. Two filtrations can model asymmetry of information on the market. We consider the price process as a canonical process on some restriction of space of $\mathbb{R}^d$-valued continuous functions on $[0, T]$. Price process represents underlying stocks and continuously traded options. Beside that we allow static position in options from a given set with given prices. One may look at the superhedging prices for an informed agent or at the market model prices induced by appropriate sets of martingale measures. Our main result is showing that the pricing–hedging duality holds for the informed agent for some class of payoffs, in a number of interesting cases. Based on joint work with Z. Hou and J. Obłój.

Cristin BUESCU, King's College London.
Title: *Funding Inclusive Valuation as Modified Option Pricing*.
Abstract: Valuation of contracts in the presence of funding costs, credit risk (default closeout), and repo borrowing/lending leads in general to nonlinear problems that are difficult to solve, even numerically, making sensitivity analysis intractable. We specialize some earlier nonlinear valuation contributions to benchmark derivatives: under simplifying assumptions that restore the linearity of the valuation operator, the valuation paradigm for these benchmark products can be cast as the familiar Black-Scholes model with dividends. In turn, this allows for a detailed valuation analysis, stress testing and risk analysis via sensitivities that would have been out of reach otherwise. Based on joint work with D. Brigo and M. Rutkowski.

Umut CETIN, London School of Economics.
Title: *Linear Inverse Problems for Markov Processes and their Regularization*.
Abstract: We study the solutions of a family of inverse problems associated to the transition function of a given Markov process X, which is linked to the solutions of some ill-posed Cauchy problem. A necessary and sufficient condition ensuring square integrable solutions is given. Moreover, a family of regularizations for the above problems is suggested. We show in particular that these inverse problems have a solution when X is replaced by a convex linear combination of X and a suitable jump process J with a Bernoulli distributed weight.

Jean-François CHASSAGNEUX, Université Paris Diderot.
Title: *Cubature Method to Solve BSDEs: Error Expansion and Complexity Control*.
Abstract: In this work, we prove error expansions for the approximation of BSDEs when using the cubature method. To profit fully from these expansions, e.g. to design high order approximation methods, we need however to control the complexity growth of the cubature method. In our work, this is achieved by using a sparse grid representation. We present several numerical results that confirm the efficiency of our new method. Based on joint work with C. Garcia Trillos.

Stefano DE MARCO, École Polytechnique.
Title: *Asymptotics and Calibration for American Options*.
Abstract: Based on a suitable representation of the exercise boundary for American options in a diffusion model, we derive an approximation of the exercise boundary close to maturity that refines the expansions known so far

in the literature. By means of the early exercise formula, this allows to derive semi-closed expressions for the price of the American Put/Call. The final product is a calibration recipe of a local volatility surface to American option data, with a complexity equivalent to the application of Dupire's formula. This is an important step when only American options, and no European options, are available on the market (as it typically happens in the case of single stocks). Based on joint work with P. Henry-Labordère.

Roxana DUMITRESCU, King's College London.
Title: *Game Options in an Imperfect Market with Default*.
Abstract: We study pricing and superhedging strategies for game options in an imperfect market with default. We extend the results obtained by Kifer in the case of a perfect market model to the case of imperfections in the market taken into account via the nonlinearity of the wealth dynamics. In this framework, the pricing system is expressed as a nonlinear g-expectation/evaluation induced by a nonlinear BSDE with jump. We prove that the superhedging price of a game option coincides with the value function of a corresponding generalized Dynkin game expressed in terms of the g-evaluation, recently introduced by Dumitrescu-Quenez-Sulem. We then address the case of ambiguity on the model, for example an ambiguity on the default probability, and characterize the superhedging price of a game option as the value function of a mixed generalized Dynkin game. We prove the existence of a cancellation time and a trading strategy for the seller which allow him/her to be super-hedged, whatever the model is. This study is introduced by the analysis of the simpler case of American option. Based on joint work with M. C. Quenez and A. Sulem.

Paul GASSIAT, Université Paris Dauphine.
Title: *On Root's Solution to the Skorokhod Embedding Problem*.
Abstract: Given a target probability measure μ, the classical Skorokhod Embedding Problem consists in finding a stopping time τ such that the stopped Brownian motion B_τ has distribution μ. In 1968, Root showed that there exists a subset of time-space such that its hitting time by Brownian motion gives a solution to this problem. Root's proof was nonconstructive, leaving open the question of how this barrier can be computed in practical cases. We will report on recent progress in this direction, applications to numerical simulations, as well as extensions to general Markov processes. Based on joint works with A. Mijatovic, H. Oberhauser and G. dos Reis.

Blanka HORVATH, Imperial College London.
Title: *Aspects of Asymptotic Expansions in Fractional Volatility Models*.
Abstract: We revisit small-noise expansions in the spirit of Ben Arous, Baudoin-Ouyang, Deuschel-Friz-Jacquier-Violante for bivariate diffusions driven by fractional Brownian motions with different Hurst exponents. A particular focus is devoted to rough stochastic volatility models, which have recently attracted considerable attention. We derive suitable expansions (small-time, energy, tails) in these fractional stochastic volatility models and infer corresponding expansions for implied volatility. This sheds light (i) on the influence of the Hurst parameter in the time-decay of the smile, and (ii) on the asymptotic behaviour of the tail of the smile, including higher orders.

Lane HUGHSTON, Brunel University London.
Title: *Lévy-Vasicek Models and the Long-Bond Return Process*.
Abstract: The classical derivation of the well-known Vasicek model for interest rates is reformulated in terms of the associated pricing kernel. An advantage of the pricing kernel method is that it allows one to generalize the construction to the Lévy-Vasicek case, avoiding issues of market incompleteness. In the Lévy-Vasicek model

the short rate is taken in the real-world measure to be a mean-reverting process with a general one-dimensional Lévy driver admitting exponential moments. Expressions are obtained for the Lévy-Vasicek bond prices and interest rates, along with a formula for the corresponding long-bond return process. We show that the pricing kernel of a Lévy-Vasicek model is uniformly integrable if and only if the long rate of interest is strictly positive. Based on joint work with D. C. Brody and D. Meier.

Arne LOKKA, London School of Economics.
Title: *Optimal Liquidation Trajectories for the Almgren-Chriss Model with Lévy Processes*.
Abstract: We consider an optimal liquidation problem with infinite horizon in the Almgren-Chriss framework, where the unaffected asset price follows a Lévy process. The temporary price impact is described by a general function that satisfies some reasonable conditions. We consider an investor with constant absolute risk aversion, who wants to maximize the expected utility of the cash received from the sale of his assets, and show that this problem can be reduced to a deterministic optimization problem which we are able to solve explicitly. In order to compare our results with exponential Lévy models, which provides a very good statistical fit with observed asset price data for short time horizons, we derive the (linear) Lévy process approximation of such models. In particular we derive expressions for the Lévy process approximation of the exponential variance-gamma Lévy process, and study properties of the corresponding optimal liquidation strategy. We look at cases where the temporary impact function follows a power-law for small liquidation speeds, but tends faster to infinity than a power-law as the liquidation speed tends to infinity. In particular, we obtain an explicit expression for the connection between the temporary impact function for the general Lévy model and the temporary impact function for the Brownian motion model, for which the optimal liquidation strategies for the two models coincide. Based on joint work with J. Xu.

Thibaut MASTROLIA, École Polytechnique.
Title: *A Tale of a Principal and Many Agents*.
Abstract: In this paper, we investigate a moral hazard problem in finite time with lump-sum and continuous payments, involving infinitely many Agents, with mean field type interactions, hired by one Principal. By reinterpreting the mean-field game faced by each Agent in terms of a mean field FBSDE, we are able to rewrite the Principal's problem as a control problem for McKean-Vlasov SDEs. We review two general approaches to tackle it: the first one introduced recently by Bayraktar, Cosso and Pham (2016) and Pham and Wei (2015, 2016) using dynamic programming and Hamilton-Jacobi-Bellman equations, the second based on the stochastic Pontryagin maximum principle, which follows the paper of Carmona and Delarue (2015). We solve completely and explicitly the problem in special cases, going beyond the usual linear-quadratic framework. We finally show in our examples that the optimal contract in the N-players' model converges to the mean-field optimal contract when the number of agents goes to infinity. Based on joint work with R. Elie and D. Possamaï.

Mohamed MRAD, Université Paris XIII.
Title: *Convergence Rate of Strong Approximations of Compound Random Maps: An Application to Utility SPDEs*.
Abstract: We consider a random map $x \to F(\omega, x)$ and a random variable $\Theta(\omega)$, and we denote by $F^N(\omega, x)$ and $\Theta^N(\omega)$ their approximations: We establish a strong convergence result, in L^p-norms, of the compound approximation $F^N(\omega, \Theta^N(\omega))$ to the compound variable $F(\omega, \Theta(\omega))$, in terms of the approximations of F and Θ. We then apply this result to the composition of two Stochastic Differential Equations through their initial conditions, which can give a way to solve some Stochastic Partial Differential Equations. Based on joint work with E. Gobet.

Eyal NEUMAN, Imperial College London.
Title: *Optimal Portfolio Liquidation in Target Zone Models and Catalytic Superprocesses.*
Abstract: We study optimal buying and selling strategies in target zone models. In these models the price is modeled by a diffusion process which is reflected at one or more barriers. Such models arise for example when a currency exchange rate is kept above a certain threshold due to central bank intervention. We consider the optimal portfolio liquidation problem for an investor for whom prices are optimal at the barrier and who creates temporary price impact. This problem will be formulated as the minimization of a cost-risk functional over strategies that only trade when the price process is located at the barrier. We solve the corresponding singular stochastic control problem by means of a scaling limit of critical branching particle systems, which is known as a catalytic superprocess. In this setting the catalyst is a set of points which is given by the barriers of the price process. For the cases in which the unaffected price process is a reflected arithmetic or geometric Brownian motion with drift, we moreover give a detailed financial justification of our cost functional by means of an approximation with discrete-time models. Based on joint work with A. Schied.

Paolo PIGATO, INRIA.
Title: *Statistical Estimation of the Oscillating Brownian Motion and Application to Volatility Modeling*.
Abstract: The Oscillating Brownian Motion is a classical, simple example of stochastic differential equation with discontinuous diffusion coefficient. It behaves like a Brownian motion which changes variance parameter each time it crosses a certain threshold. We consider here the problem of estimating the parameters of such process from discrete observations. Using some techniques based on approximations of quadratic variation and local time, we propose an algorithm for which we prove consistence and a central limit theorem giving the rate of convergence. Taken as a model for volatility of financial assets, the Oscillating Brownian Motion is a simple way to account of volatility clustering and leverage effect. We apply our algorithm to financial time series, find consistent results, and compare them with other regime switching models. Based on joint work with A. Lejay.

Sergio PULIDO, ENSIIE.
Title: *The Jacobi Stochastic Volatility Model*.
Abstract: We introduce a novel stochastic volatility model where the squared volatility of the asset return follows a Jacobi process. It contains the Heston model as a limit case. We show that the the joint distribution of any finite sequence of log returns admits a Gram–Charlier A expansion in closed-form. We use this to derive closed-form series representations for option prices whose payoff is a function of the underlying asset price trajectory at finitely many time points. This includes European call, put, and digital options, forward start options, and forward start options on the underlying return. We derive sharp analytical and numerical bounds on the series truncation errors. We illustrate the performance by numerical examples, which show that our approach offers a viable alternative to Fourier transform techniques. Based on joint work with D. Ackerer and D. Filipović.

Daniel SCHWARZ, University College London.
Title: *Integral Representation of Martingales in Mathematical Finance*.
Abstract: In this talk we shall present recent results concerning a class of integral representation theorems for martingales which lie at the heart of two fundamental problems in mathematical finance: the completion of financial markets with derivative securities and the existence of partial Radner equilibria. Some popular examples and open problems will be discussed.

ABSTRACTS OF THE POSTER PRESENTATIONS

Eduardo ABI JABER, Université Paris Dauphine.
Title: *Stochastic Invariance of Closed Sets with Non-Lipschitz Coefficients*.
Abstract: We provide a new characterization of the stochastic invariance of a closed subset of $\mathbb{R}^d$ with respect to a diffusion, i.e. we give necessary and sufficient conditions on the drift and the covariance matrix under which the diffusion remains in the domain. We extend the well-known inward pointing Stratonovich drift condition to the case where the diffusion matrix can fail to be differentiable: we only assume that the covariance matrix is. In particular, our result can be directly applied to construct affine and polynomial diffusions on any arbitrary closed set. We exemplify our result by providing a generic tractable characterization for the stochastic invariance in a two dimensional set-up. Based on joint work with B. Bouchard and C. Illand.

Matteo BASEI, Université Paris Diderot.
Title: *Coordination of Centralized and Distributed Generation*.
Abstract: Consumers satisfy their electricity demand by self-production (solar panels) and centralized production (energy companies). Both companies and consumers are interested in characterizing the production strategies which minimize the costs and satisfy some properties; in particular, consumers need low-variance policies so as to have a stable flux of energy, whereas companies face carbon taxes and a penalty in case of under/overproduction. We address three problems: namely, we consider the point of view of a representative consumer, a representative energy company and a social planner. We deal with McKean-Vlasov control problems with stochastic coefficients and provide explicit formulas for the optimal strategies of each problem; finally, we compare the results and look for an electricity price such that the optimal controls asymptotically coincide in mean. Based on joint work with R. Aïd, I. Ben Tahar and H. Pham.

Jacopo CORBETTA, École des Ponts ParisTech.
Title: *A New Approach to Backtesting and Risk Model Selection*.
Abstract: Backtesting risk measures represent a challenge and complex methods are often required. In this paper, we propose a new framework for backtesting that can be applied to every risk measures. We base our approach on the formalization of the concept of level of coverage associated with the risk model. Thus, we propose two simple hypothesis tests based only on results of probability theory without requiring any approximation or simulation. In addition, within this new framework, we introduce a methodology for selecting the best performing model among all the existing alternatives. This proposal adds value to the current state of the art, since, using the traditional loss function approach, any comparison among forecasting outcomes of different risk models appeared to be meaningless. Based on joint work with I. Peri.

Côme HURÉ, Université Paris Diderot.
Title: *Optimal Placement of Orders in Market Making Strategies*.
Abstract: We consider a Market Maker who stands ready to buy and sell stocks on a regular and continuous basis at quoted prices. We want to find optimal strategies that maximize her wealth at a terminal time. We use the theory of Markov Decision Processes to show the existence of optimal strategies. We characterize them as solutions of Bellman equations. Finally, we use Markovian quantization methods to compute efficiently the optimal strategies. Based on joint work with F. Abergel and H. Pham.

David KRIEF, Université Paris Diderot.
Title: *An Asymptotic Approach for the Pricing of Options on Realized Variance*.

Abstract: We propose a method for the approximate pricing of options on realized variance in stochastic volatility models where the instantaneous volatility process is a diffusion process with general drift coefficient and constant diffusion coefficient. This can be seen as an extension of the Stein-Stein model. The pricing problem consists in calculating the price of a call option on realized variance which is the time integral of the squared volatility. Our method for computing this price is based on the asymptotic expansion of the density of the integrated variance in the spirit of Deuschel et al. (2014). We then integrate the payoff of the option against the asymptotic expression of the density to obtain an approximation of the option price using Laplace's method. Based on joint work with Z. Grbac, A. Gulisashvili and P. Tankov.

Michael KUSNETSOV, London School of Economics.
Title: *Systemic Risk in Interbank Networks with Multiple Maturities*.
Abstract: We consider the problem of systemic risk assessment in interbank networks in which interbank liabilities can have more than one maturity. In particular, we allow for both short-term and long-term interbank liabilities. We develop a clearing mechanism for the interbank liabilities to deal with default of one or more market participants. Our approach generalises the clearing approach proposed in Eisenberg & Noe (2001) for the single maturity setting. We also describe novel effects that arise as soon as more than one maturity date is considered and discuss implications for systemic risk assessment. Based on joint work with L. Veraart.

Hao LIU, Imperial College London.
Title: *Optimal Liquidation in a Level-I Limit Order Book for Large-Tick Stocks*.
Abstract: The purpose of this paper is to formulate and solve a stylized optimal liquidation problem for a certain agent who wants to sell a pre-specified quantity of large-tick stocks over a fixed intra-day trading window in a LOB (limit order book) where the price-time priority mechanism is applied. Information available to this agent only contains the historical order flows and states of the LOB at the best prices (i.e. the "Level-I" data). In particular, we are mostly interested in how different trading conditions (e.g. LOB state, inventory position, time to maturity) impact the agent's decisions. In order to achieve this, we first build up a "Level-I" LOB model according to R. Cont (2010), which describes the trading environment whose dynamics are driven by the general market participant order flows and the exogenous information. We then consider a risk-neutral agent who tries to maximize the expected terminal wealth by selling a fixed amount of stock within a determined finite-time horizon in this LOB. Under simplifying assumptions, the agent's trading procedure is then formulated through a (stationary) semi-Markov decision process within a finite horizon with the optimality is among a certain class of deterministic stationary policies. Furthermore, the semi-Markov kernel is calculated through applying the Laplace's method. And the optimal strategy is obtained by approximated dynamic programming in order to overcome the "curse of dimensionality".

Maxime MORARIU-PATRICHI, Imperial College London.
Title: *Limit Order Book Modelling with Interacting Point Processes*.
Abstract: My aim is to introduce the research objective and approach that I am currently following in the area of limit order book (LOB) modelling. After explaining the LOB mechanism, I will discuss the use of point processes to model the interaction between the price moves, the order flow, and the LOB's state. First, I will recall the empirical linear relation between the price moves and the order flow imbalance that was found by Cont, Kukanov & Stoikov (2013). This result can be used to reduce the dimensionality of the modelled objects by splitting the order flow into two categories of events. Second, I will review two main trends in the literature (Markov models and models that are based on Hawkes processes) and discuss the limitations of each approach. Third, I will present empirical results suggesting that one should keep track of the LOB's state in order flow

models. More precisely, I will show a new type of interaction between the order flow and the queue imbalance that we are currently trying to model. Based on joint work with M. S. Pakkanen.

Udomsak RAKWANGWAN, King's College London.
Title: *Option Portfolio Optimization and Valuation Under Transaction Costs*.
Abstract: We develop a convex optimization model for optimal investment and contingent claim valuation in options markets where price quotes have bid-ask spreads and finite quantities. Using computational techniques of convex optimization, we illustrate the approach numerically with forward and option contracts on S&P 500 index. Our results quantify the dependence of optimal portfolios and option valuations on investors' beliefs (the probabilistic model), risk preferences (utility function), financial positions (initial wealth and liabilities), and market conditions (price quotes, quantity constraints, lending and borrowing rates).

Guillaume SALL, Université Pierre et Marie Curie.
Parareal Methods and Applications in Finance.
Abstract: We aim to show the interest of the parareal method (based on the work of Y. Maday and J. L. Lions) for the approximation of discrete problems based on backward scheme using least-squares regression i.e. the Longstaff-Schwarz algorithm, approximation of a backward stochastic differential equations. We propose a parallelization of the time discretization of the backward dynamic programming principle. It allows us to approximate simultaneously the solution at several time-steps. Here, we analyze this algorithm and the convergence of the parareal scheme. We also give some performance results for the pricing of an American option. Because of its parallel scalability the method is well suited to fast evaluation of CVAs. Based on joint work with G. Pagès and O. Pironneau.

Luciane SBARAINI BONATTO, King's College London.
Title: *Convex Stochastic Optimization Model for Oil Derivatives Trading*.
Abstract: We developed a model for oil derivatives portfolio optimization. The model is easily adjusted to the trader's views, risk preferences, and financial positions, as well as to market conditions including available quotes and various constraints. The model is convex and solvable with the efficient techniques of numerical convex optimization. We illustrate the results with data from crude oil and refined products most liquid derivative contracts traded on New York Mercantile Exchange (NYMEX) and Intercontinental Exchange (ICE).

Fangwei SHI, Imperial College London.
Title: *The Randomized Heston Model*.
Abstract: Implied volatility is one of the most important data in financial markets. A number of stochastic volatility models have been proposed to understand its dynamics. Classical stochastic volatility models (driven by Brownian motion) with continuous paths can effectively fit the volatility smile/surface for most maturities. However, such kind of models fails to reproduce the steep volatility smile generated from the small-maturity market data (the so-called "short-time explosion" feature). To solve this issue, we propose a randomized version of the Heston model, assuming that the starting point of the variance process is a (continuous) random variable. In this randomized environment, we study the small-time and large-time behaviour of the implied volatility, and show that our model generates a short-time smile much steeper than in the standard Heston model. We relate the speed of explosion of the short-time volatility smile to the right-tail behaviour of the initial distribution, and in particular show that an explosion rate in $[0, 1/2]$ for the squared implied volatility can be obtained by a suitable choice of randomization. Based on joint work with A. Jacquier.